Answers

1. $A = 2\pi r + \pi$, $r \geq 0$
2. (a) $A(1) \approx 30$, $A(4) \approx 40$, (b) $A(x) = 40$ for $x \approx 3$ and $x \approx 4$. $A(x) = 60$ has no solution with $1 \leq x \leq 6$.
   (c) $A(2) \approx 26$, $[A(2)]^2 \approx 26^2 = 676$ $A(2^2) = A(4) \approx 40$, $[A(2)]^2$ is greater.

3. If A is a function, then A(2) is its value at 2. If A is a constant, then A(2) is A multiplied by 2.

4. $p(1) = 18$, $p(2) = -6$, $p(3) = \sqrt{2}$

5. Possible answers: (a) She is stopped from about t = 40 to t = 55, which is about 15 seconds. (b) At about t = 110 (c) From about t = 70 to t = 115 (d) From about t = 7 to t = 27, She goes about 1/6 mile.

6. (a) Least inrease in number of PAC's = 111 (from 1986 to 1988), Greatest increase = 898 (from about 1978 to 1980) (b) Least increase in contributions = $10,045,000 (from 1974 to 1976), Greatest increase = $31,403,000 (from 1980 to 1982) (c) $20,604 per PAC in 1974 and $34,653 per PAC in 1988(rounded to the nearest dollar)

7. (a) 10Q(10) = 210 (b) Q(6 + 3 + 1) = 21 (c) Q(100/20) = 7 (d) Q(10)/Q(5) = 3 (e) Q(Q(5) + 3) =21

8. (a) Change = 4300 – 3500 = 800 cigarettes per person, Relative change = 800/3500 = 0.23 (b) Change = 3300 – 4300 = -1000 cigarettes per person, Relative change = -1000/4300 = -0.23 (c) C(1953) = 3700 cigarettes per person, Error = |3700 - 3500| = 200 cigarettes per person, Relative error = 200/3500 = 0.057 (d) The first graph does not show relative change well because the C-axis is not shown for $0 \leq C \leq 3300$.

9. $P(100) = 3P(50) = 3(2P(1)) = 6P(1)$, Relative change $= \dfrac{P(100) - P(1)}{P(1)} = 5$

10. A(1960) = 100(1610)/(1610 + 18581) = 7.97, A(1965) = 14.04, A(1970) = 30.24, A(1975) = 38.44, A(1980) = 45.84, A(t) is the percentage in year t of the km traveled by Japanese in private cars, trains, and busses that were traveled in private cars.

11. Possible answer: Set F(0)=1. Then F(1) = 4F(0) = 4, F(2) = 4F(1) = 16, F(3) = 4F(2) = 64, F(4) = 4F(3) = 256, and F(5) = 4F(4) = 1024.

12. (a) $\dfrac{4-\pi}{4}$ X100 = 21.5% (b) $\dfrac{\pi - 2}{\pi}$ X 100 = 36.3% (c) Circumscribed area = 2(inscribed area)

13. $y = 36/x^2 - 25$, $y = 20 - 3x^2$, $y = -24/x$, $y = x^3 - 5$.

14. x^{-1} dominates x^{-2} for large positive and large negative. x^{-2} dominates x^{-1} for small positive and large negative x.

15. (a) x^3 is the dominant term in the numerator and $7x^2$ is the dominant term in the denominator when x is a large positive or negative number. (b) –5 is the dominant term in the numerator and 3x is the dominant term in the denominator when x is close to zero.

16. (a) figure 12 (b) figure 11, b is positive (c) figure 13, a is positive

17. Figure 15, b is negative since $y \to -\infty$ *as* $x \to 0$

18. Figure 17, Both a and b are positive since y = a when x = 0 and $y \to +\infty \ as \ x \to +\infty$

19. Figure 14, a is positive since $y \to a \ as \ x \to \pm\infty$, b is negative because $y \to -\infty \ as \ x \to 0^+$

20. 200 miles + (t hours)(60$\frac{\text{miles}}{\text{hours}}$) = 200 + 60t miles

21. $\frac{1}{0.05 \text{ gallons/mile}} = 20 \frac{\text{miles}}{\text{gallon}}$

22. V = 20 – (s miles0 (0.05 $\frac{\text{gallons}}{\text{mile}}$)=20 – 0.05s gallons The tank is empty when 20 – 0.05s = 0, which is at 400 miles, so the domain of V as a function of s is the interval $0 \le s \le 400$.

23. s = 60t miles. Since s = 400 at t = 6 2/3, the domain of s a function of t is $0 \le t \le 6\ 2/3$.

24. (a) V = 20 – 3t for $0 \le t \le 6\ 2/3$ (b) -3 gallons per hour (c) The volume of gas in your tank decreases 3 gallons every hour.

25. -6

26. f(x) = 10 + 50(x – 3)

27. -1/3

28. The average rate of change of Z(x) with respect to x from x = 0 to x = 5 is $\frac{200 - 100}{5 - 0} = 20$. Z(x) = 20x + 100

29. (a) w = 1.58V (b) 1.58 grams per ml

30. (a) L = 100 + 0.093(T – 50) (b) L = 100.5 at T= 55.38

31. (a) p = 0.44h + 14.7 (b) p(36198) = 0.44(36198) + 14.7 = 15941.82 pounds per square inch.

32. (a) s = -15t + 300 (b) t = 20 sec (c) -15 meters per sec

33. (a) v(t) = 32t + 5 (b) 1 second (c) After 3 seconds

34. One answer: 1/25 liters per minute per liters per minute

35. (a) $A = \pi(0.6)^2 - \pi(0.5)^2 = 0.11\pi$ square inches. (b) $V = 0.11\pi L$ cubic inches (c) C = 0.05V so $V = 0.0055\pi L$. The rate of change of cost with respect to length is $0.0055\pi = 0.0173$ dollars per inch.

36. The rate of change of the perimeter of the square with respect to width is 4 and is greater than the rate of change π of the circunference with respect to the diameter.

37.

t hours	0	1	2	3	4	5
s(t) miles	100	131	168	217	284	375

38. In the four hours from t = 1 to t = 5, the plane travels 375 – 131 = 244 miles. Its average velocity is $\frac{244\text{km}}{4\text{ hours}}$ = 61 miles per hour.

39. The plane's average velocity for $0 \le t \le 4$ is $\frac{s(4) - s(0)}{4 - 0} = \frac{284 - 100}{4}$ = 46 miles per hour. The secnat line passes through the points (0,100) and (4,284) on the graph.

40. As you zoom in, the curve looks more and more like a line.

41. The plane's average velocity for $2.999 \le t \le 3$ is $\frac{s(3) - s(2.999)}{3 - 2.999}$ = 56.991 miles per hour. Its average velocity for $3 \le t \le 3.001$ is $\frac{s(3.001) - s(3)}{3.001 - 3}$ = 57.009 miles per hour.

42. Average velocity = $\frac{s(5) - s(1)}{5 - 1}$ = 10 yards per minute.

43. Average velocity for $1.99 \le t \le 2$ = $\frac{s(2) - s(1.99)}{2 - 1.99}$ = 12.56281

Average velocity for $2 \le t \le 2.01$ = $\frac{s(2.01) - s(2)}{2.01 - 2}$ = 12.43781

Average velocity for $1.99 \le t \le 2.01$ = 12.50031. The velocity at t = 2 is approximately 12.5 yards per minute.

44. (a) Average rate of chane for $2 \le x \le 5$ = $\frac{W(5) - W(2)}{5 - 2}$ = 2/3

(b) Average rate of change for $1 \le x \le 3$ = $\frac{W(3) - W(1)}{3 - 1}$ = 0

(c) The average rate of chnage for $2.50 \le x \le 2.51$ is the slope 2 of the line from (2,1) to (3,3).

45. (a) Average velocity $0 \le t \le 3$ = 30 miles per hour (c) She speeds up (d) 30 miles per hour.

46. (a) s(t) = 10/t equals 5 at t = 2 (b) s(t) = 10/t equals 1 at t= 10 (c) $\frac{s(10) - s(2)}{10 - 2}$ = -1/2 (e) No, since s(t) > 0 for t >0.

47. (a) She starts 100 miles east of Reno, drives east about 4 hours, when she is about 440 miles from Reno. Then she drives west for about 1 hour and ends up about 350 miles east of Reno. (b) average velocity toward the east for $0 \le t \le 4$ = 86 miles per hour, avergae velocity toward the east for $4 \le t \le 5$ = -94 miles per hour (c) velocity toward the east at t= 2 is about 118 miles per hour, velocity toward the east at t = 4.5 is about -89.6 miles per hour

48. (a) T = 4 (b) V(4) = 48 gallons (c) at t = 1, Possible answers: $\frac{V(1) - V(0.99)}{0.01}$ = 18 gallons pr hour ans $\frac{V(3) - V(2.99)}{0.01}$ = 6 gallons per hour

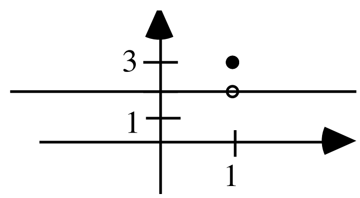
49. (a) 1000 feet per minute (b) = -.0006 pounds per square inch per foot (c) = -0.6 pounds per square inch per minute (d) The answer to part (c) is the product of the answers to (a) and (b): $1000\,\frac{\text{feet}}{\text{minute}}(-0.0006\,\frac{\text{pounds per square inch}}{\text{foot}}) = -0.6\,\frac{\text{pounds per square inch}}{\text{minute}}$

50. S(2) = 14 , $\lim_{x\to 2^+} S(x) = \lim_{x\to 2^+} x^8 = 2^8 = 256$, $\lim_{x\to 2^-} S(x) = \lim_{x\to 2^-} \sqrt{x+2} = \sqrt{2+2} = 2$, $\lim_{x\to 2} S(x)$ does not exist.

51. The values suggest 1.7.

52. $\lim_{x\to 8} f(x) = f(8) = 5$

53. Many different answers. One possible answer:



54. One answer: $h(x) = \begin{cases} 1/x^2 & \text{for } x \neq 0 \\ 2 & \text{for } x = 0 \end{cases}$

55. $\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} x^3 + x = 10$, $\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} 10 = 10$, $\lim_{x\to 2} g(x) = 10$, $\lim_{x\to 10^-} g(x) = \lim_{x\to 10^-} 10 = 10$, $\lim_{x\to 10^+} g(x) = \lim_{x\to 10^+} 1 + \sqrt{x+7} = 1 + \sqrt{17}$, $\lim_{x\to 10} g(x)$ does not exist. g(x) is continuous in$(-\infty,10]$ and $(10, \infty)$.

56. $\lim_{x\to 10^-} Z(x) = \lim_{x\to 10^-} x^2 + 900 = 1000$ and $\lim_{x\to 10^+} Z(x) = \lim_{x\to 10^+} x^3 = 1000$, so $\lim_{x\to 10} Z(x) = 1000$, $\lim_{x\to 20} Z(x) = \lim_{x\to 20} x^3 = 8000$

57. Continuous at x = -1 because f(-1) = 0 and $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) = 0$, *so* $\lim_{x\to -1} f(x) = 0$, which are all equal. It is discontinuous at x=1 because $\lim_{x\to 1^-} f(x) = 6$ and f(1) = 7, which are different numbers.

58. $\lim_{x\to 2^-} Z(x) = 4k$, $\lim_{x\to 2^+} Z(x) = 8$, The two-sided limit exists if k = 2.

59. g(-1) = -a +b, $\lim_{x\to -1^-} g(x) = -1$, and $\lim_{x\to -1^+} g(x) = -a + b$, so g(x) is continuous at x = -1 if and only if -1 = -a + b. Also, g(1) = 3, $\lim_{x\to 1^-} g(x) = 3$, and $\lim_{x\to 1^+} g(x) = a + b$, so g(x) is continuous at x = 1 if and only if 3 = a + b. Solving these 2 equations for a and b yields a = 2 and b = 1.

60. h(-2) = 2 , $\lim_{x\to -2^+} h(x) = 4a + b$, and $\lim_{x\to -2^-} h(x) = 2$, so h(x) is continuous at x = -2 if and only if 4a + b = 2 . Also h(0) = 6 and $\lim_{x\to 0^-} h(x) = b$, so h(x) is continuous from the left at x = 0 if and only if b = 6 so a = -1.

61. Since $\sqrt{x^4 + 3} = 2$ at x = 1, the derivative is the limit of $\frac{\sqrt{b^4 + 3} - 2}{b - 1}$ *as* $b \to 1$. Use this formula to predict the derivative of $\sqrt{x^4 + 3}$ at x =1 is 1.

62. The value of T(x) at 9.75 is approximately equal to the value 7 + 4(t – 10) (the equation of the tangent lineat x = 10) when t = 9.75 or 7 + 4(9.75 – 10) = 6.

63. (a) $\frac{s(2) - s(0)}{2 - 0} = 1$ ft per sec (b) $\frac{ds}{dt} = t$, $\frac{ds}{dt}(1) = 1$ ft per sec

64. Average rate of change for $1 \le t \le 4 = -5$ degrees per hour. $T'(t) = -32/t^3$, $T'(2) = -4$ degrees per hour. The instantaneous rate of change at t = 2 is greater.

65. (a) $V'(t) = -30t^2$, $-30t^2 = -270$ at t = 3.

66. (a) $W'(w) = 3w^2$, $W'(3) = 27$ cubic yards per yard. (b) average rate of change for $0 \le w \le 3 = 9$ cubic yds per yd. average rate of change for $1 \le w \le 3 = 13$ cubic yds per yd. Less

67. (a) She has walked 7 miles 3 hours after dawn. (b) She is walking 3 miles per hour 3 hours after dawn. (c) She is walking in the opposite direction at 0.5 miles per hour 5 hours after dawn. (d) Her average velocity is 2 miles per hour during he 5 hours after dawn.

68. (a) The median price of new houses at the beginning of 1991 was $97,000 greater than at the beginning of 1970. (b) The median price of new houses at the beginning of 1991 was 1.25 times the median price of existing houses. (c) The median price of existing houses at the beginning of 1991 was 5 times the median price of existing houses at the beginning of 1970. (d) At the beginning of 1991 the median price of existing houses was increasing. (e) At the beginning of 1991 the rate of change of the median price of new houses was decreasing at 90% of the rate at which the median price of existing houses was increasing.

69. (a)There were 38 mg of tar in the average cigarette manufactured in the US in 1957.
(b) The average amount of tar in US cigarettes was decreasing 6 mg per year in 1957.
(c) There were 24 more mg of nicotine in the average cigarette in 1980 than in 1957.
(d) The rate of increase of the amount of nicotine in the average cigarette in 1980 was 1/3 what it was in 1957.

70. (a) v(4) = 116 knots (b) $\frac{dv}{dh}(6) = -0.5$ knots per 1000 feet, $\frac{dg}{dh}(6) = -0.2$ gallons per hour per 1000 feet (c) $\frac{dg}{dh}(12) = 0.81 \frac{dg}{dh}(4)$ gallons per hour per 1000 feet.

71. (a) From 4 meters with an upward velocity of 39.2 meters per sec. (b) v = 0 when t = 4 (c) 82.4 meters

72. (a) t = 2 (b) v > 0 for 0 < t < 2.

73. $v(t) = 120 - 120t^{1/2}$ (a) v90.25) = 60. The car is traveling east at 60 miles per hour. (b) v(t) = 0 when t = 1, s(1) = 100 miles.

74. (a) The tangent line passes through the points with approximate coordinates (3,15) and (4,23) so P '(3) =8. (b) x is approximately 1.7 and 4.1 (c) Draw a line of slope 5 and find the points where the tangent line is parallel to it: x about 2.2 or 3.6.

75. J '(x) is approximately 0 at x = 0. Its greatest and lest values for x are about 1.5 at x = 1.3 nd -1.5 at x = -1.3. Its values at x = ±5 are approximately ±0.3.

76. (a) The graph of P '(x) is in figure 29. P '(x) is negative for negative x, 0 at x = 0, and positive for positive x. (b) The graph of Q '(x) is in figure 30. Q '(x) is negative for nonzero x and zero at x = 0. (c) The graph of R '(x) is in figure 28. R '(x) is positive for nonzero x and 0 and x = 0.

77. $\frac{dQ}{dx}(10) \approx 16$ or 14.5 or 15.25, depending on which kind of difference quotients used. $\frac{dQ}{dx}(10.35) \approx 12.5$

78. T(500) ≈16 degrees Celsius , $\frac{dT}{dh}(500) \approx -0.0045$ degrees C per foot, T(1500) ≈ 16 degrees C, $\frac{dT}{dh}(1500) \approx 0.01$ degrees C per ft, T(7000) ≈ 17.5 degrees C, $\frac{dT}{dh}(7000) \approx -0.0015$ degrees C per ft. (Different estimates give different answers)

79. Starts at about 30 on the vertical scale, decreasing to 0 at about t = 1.2. Reaches a minimum of about -15 at about t = 2.8 and then increase and is 0 at about t = 5.1. at t = 8 the velocity is about 10 feet per min.

80. (a) $\frac{dN}{dt}(1979) = -0.01$ or -0.02 or -0.015 million farms per year depending on the type of difference quotient used. (b) $\frac{dA}{dt}(1984) \approx -5.33$ million acres per year. (c) $\frac{A(1976)}{N(1976)} \approx 422$ acres per farm, $\frac{A(1991)}{N(1991)} \approx 467$ acres per farm.

81. (a) The percent of water W in a piece of wood should increase as the humidity increases, causing W '(t) to be positive. (b) W(69) ≈ W(70) + W '(70) (69 – 70) = 10.45%

82. (a) P(t) was a minimum at t ≈ 1969 when $\frac{dP}{dt}(t) = 0$ (b) $\frac{dP}{dt}(t)$ was a minimum at t ≈ 1965 (c) $\frac{dP}{dt}(1965) \approx -2.2\%$ per year.

83. (a) $\frac{dL}{dx}(0) \approx 0.9$ thousand pounds per degree, $\frac{dL}{dx}(13) \approx -1$ thousand pounds per degree (b) max lift ≈ 15000 pounds at t ≈ 11, L ' = 0 at the max

84. $A = 4\pi r^2$, $r = \frac{1}{2}\pi^{-1/2}A^{1/2}$, $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi^{-1/2}A^{3/2}$, $\frac{dV}{dA} = \frac{1}{4}\pi^{-1/2}A^{1/2}$

85. $V = w^3$, $w = V^{1/3}$, $S = 6w^2 = 6V^{2/3}$, $\frac{dS}{dV} = 4V^{-1/3}$

86. $s(t) = 60t + \frac{1}{10}t^5$, $v(t) = 60 + \frac{1}{2}t^4$, $a(t) = 2t^3$, a(t) =16 at t= 2. s(2) = 123.2 km, v(2) = 68 km per hour

87. (a) max velocity ≈ 3.4 ft per min; min velocity ≈ 0.2 ft per min (b) Figure 36. a(t) =0 near t = 1.4 when v(t) is a min and near t = 2.45 when v(t) is a max. (c) k = 5 since the max derivative of v(t) ia bout 5 near x = 2.

88. C"(t) < 0 but C '(t) > 0, where c(t) is the cost of health care at time t.

89. The rate of increase of the amount of emissions dropped, but not the emissions,

90. (a) The population of women at time t is W(t) = F(t)P(t), W(1990) = F(1990)P(1990) = 127.6 million (b) $\frac{dW}{dt}(1990) = F(1990)\frac{dP}{dt}(1990) + P(1990)\frac{dF}{dt}(1990)$ = 1.7 million per year. (c) At the beginning of 1990, the rate of change of men was = 3.5 - 1.7 = 1.8 million per year.

91. (a) D(1985) ≈ 1850 billion dollars, P(1985) ≈ 240 million people, The points (1980,600) and (1985,1850) are on the approx. tangent line on the debt graph. The points (1970, 180) and (1985,240) are on the approx. tangent line on the population graph. D '(1985) ≈ 250 billion dollars per year, P '(1985) ≈ 4 million people per year. (b) Debt per person = D/P ≈ 1850/250 = 7.4 thousand dollars per person. Rate of change of debt per person = the derivative of D/P= $\frac{PD' - DP'}{P^2}$ = 0.913 thousand dollars per person per year.

92. (a) $\frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}$, (b) $4\pi(3^2)(1/2) = 18\pi$ cubic inches per sec

93. P'(5) = 18

94. 3/25

95. $\frac{-35}{2}(6^{5/2})$

96. Possible answers: A(2) ≈ 2.5, A '(2) ≈ -0.6, B(2) ≈ 1.5 , B '(2) ≈ 1, [AB] at x = 2 is ≈ 3.75, [A/B] at x = 2 is ≈ 1.7. $\frac{d}{dx}(AB)$ at x = 2 is ≈ (2.5)(1) + (1.5)(-0.6) = 1.6. $\frac{d}{dx}(A/B)$ at x = 2 is ≈ -1.5, using quotient rule.

97. $R'(s) = \frac{QP' - Q'P}{Q^2}$, R' (1) = 19/2

98. Let f(t) be the size of the average farm and N(t) the number of farms at time t. Total acreage = A(t) = N(t)f(t). A(1991) = 980.7 million acres. A '(1991) = Nf ' + N 'f =(2.105)(7) + (-0.035)(467) = -1.61 million acres per year. The total acreage was decreasing 1.61 million acres per year.

99. A = wL. A' = wL' + Lw ' = -9 square meters per second. The area is deceasing 9 square meters per second.

100. Number held for drug offences = D(t) = 0.01 P(t)N(t). D(1993) ≈ 0.60(900) = 540 thousand. D(1979) ≈ 0.25(300) = 75 thousand. D(1993) - D(1979)≈465 thousand. (b) D '(1988) = 0.01P'N + 0.01PN'≈ (0.45)50 + 6(4.125) = 47.25 thousand per year.

101. Let T(t) = Total Cost, C(t)= Cost per person, and P(t) = population. T(t) = C(t)P(t), T '(1990) = $71760 million per year.

102. Possible answers: Debt per card = B(t) = D(t)/N(t). D(1988) ≈ 120000 million dollars. N(1988) ≈ 125 million. B(1988) ≈ 980 dollars per card. D '(1988) ≈ 21000 million dollars per year. N '(1988) ≈ 8.3 million cards per year. B '(1988) ≈ $\frac{125(21000) - 120000(8.3)}{(125)^2}$ = 104.03 dollars per card per year.

103. $\frac{S(t)}{P(t)}$ is the number of pairs of shoes purchased per year per person. $\frac{d}{dt}(S/P)$ is the rat of change of the number pf pairs of shoes purchased per year per person. Possible answers: S(1982) ≈ 830 and P(1982) ≈ 231. So, S/P ≈ 3.59 pairs of shoes per year per person. S '(1982) ≈ 125 and P' (1982) ≈ 2.4. $\frac{d}{dt}(S/P)$ = 0.504 pairs per year per person per year.

104. -24

105. 2/9

106. -7/4 since G '(2) = 2 and G(2) = 11 since y = 7 + 2x is the tangent line at x = 2

107. $\frac{1}{4}(W'(2) - 6) = 4$ so W '(2) = 11.

108. 5/3

109. 84

110. (a) $(3 - t)^2 = 4$ at t = 1 (b) $(3 - t)^2 = 0$ at t = 3 (c) V '(1) = -4. Water is flowing out at the rate of 4 gallons per hour.

111. 900 cubic mm per day

112. (a) 4 grams per cm (b) $\frac{d\rho}{dT} = -160L^{-2}\frac{dL}{dT}$ = -0.001 gm per cm per degree

113. $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$, $\pi r^2 = 16$ for $r = 4/\sqrt{\pi}$, $(-3) = \pi\left(\frac{8}{\sqrt{\pi}}\right)\left(\frac{dr}{dt}\right)$. So $\frac{dr}{dt} = -\frac{3}{8\sqrt{\pi}}$. The radius is decreasing $\frac{3}{8\sqrt{\pi}}$ inches per minute.

114. (a) V(5) = 654.5 cubic inches. $V'(h) = 20\pi h - \pi h^2$. So, $V'(5) = 75\pi$ cubic inches per inch. (b) $\frac{dV}{dt} = 20\pi h\frac{dh}{dt} - \pi h^2\frac{dh}{dt}$. So at the time in question $\frac{dV}{dt} = 450\pi$ cubic inches per min.

115. (a) $\frac{200}{t + 20} = 4$ at t = 30 sec. (b) $a(t) = v'(t) = -200(t + 20)^{-2} = -0.008$ at t = 30.

116. $\frac{dD}{dt} = \frac{1}{15}v\frac{dv}{dt} = \frac{1}{15}v(t)\,a(t)$. At the time undr consideration, $\frac{dD}{dt}$ = 1666 2/3 pounds per hour.

117. $\frac{dA}{dt} = w\frac{dh}{dt} + h\frac{dw}{dt}$ One answer: $w(30) \approx 80$, $h(30) \approx 50$, $\frac{dw}{dt}(30) \approx 2$, $\frac{dh}{dt}(30) \approx -5$. So, $\frac{dA}{dt}(30) \approx -300$ square meters per minute.

118. (a) Concave down in $(-\infty,-1)$ and $(4,\infty)$. Concave up in $(-1,4)$ (b) Inflection points at $x = -1,4$.

119. (a) Concave up in (2,7) and concave down in (0,2) and (7,12) (b) The inflection points are at $x = 2,7$.

120. $g''(x) = x(x - 1)^2(x - 2)^3$ The graph of g(x) is concave up in $(-\infty,0)$ and $(2,\infty)$, is concave down in (0,1) and (1,2), and has inflection points at $x = 0,2$.

121. $K'(x) = x(4 - x)$. $K''(x) = 2(2 - x)$. K(x) is increasing in [0,4], decreasing in $(-\infty,0]$ and $[4,\infty)$ and has a local max at $x= 4$ and a local min at $x = 0$. The graph is concave up in $(-\infty,2)$ and concave down in $(2,\infty)$ and has an inflection point at $x = 2$.

122. (a) The max rate of flow into the tank is 15 gallons per hour at $t = 0$ and $t = 6$. The max flow out of the tank is 5 gallons per hour is at $t =3$. (b) Increasing in [0,2] and [4,8]. Decreasing in [2,4] (c) $V'(t)$ is increasing in [3,6]. $V'(t)$ is decreasing in [0,3] and [6,8]. (d) V(t) has a local max at $t = 2$ and a local min at $t = 4$.

123. $y' = 3x^2 - 3a = 0$. y has no local max or min if $a \le 0$. For $a > 0$: y(x) has a local max of $2a\sqrt{a}$ at $x = -\sqrt{a}$ and a local min of $-2a\sqrt{a}$ at $x = \sqrt{a}$.

124. $G'(10) < \frac{G(10) - G(0)}{10} G'(0)$.

125. $P = s + 2w$ feet . If $P = 40$, then $s = 40 - 2w$. (c) w cannot be negative or be greater than half the length of available fence.

126. (a) $A = w(40 - 2w)$ square feet for $0 \le w \le 20$. (b) The max area is when $w = 10$ feet, and $A = 200$ square feet.

127. For $w = 10$: $s = 20$ feet and $A = ws = 200$ square feet.

128. (a) If the volume is 20 cubic meters, then $h = 80$. Each of the top and bottom has area 1/4 square meters and costs \$1. Each of the four sides has area 40 square meters and costs \$80. The total cost $C = 2(1) + 4(80) = \$322$.

129. (a) $C(w) \to \infty$ as $w \to \infty$ and as $w \to 0^+$. The box would cost a great deal if you made it extremely wide or extremely narrow. (b) The minimum cost is approx. \$111.40 for a box with $w \approx 2.15$ feet.

130. Since the box that costs the least has width $10^{1/3}$, its height is $20/w^2 = 4.30887$ meters and it costs $C(w) = 8w^2 + 160/w = 111.40$ dollars.

131. (a) $A = wh$, $A = 7$ so $h = 7/w$. $P = 2w + 2h = 2w + 14/w$. (b) $P = 100$, $h = 50 - w$ and $A = w(50 - w)$, defined for $0 \le w \le 50$.

132. (a) $V = w^2h$ and $A = 4wh + 2w^2$ and for $A = 24$, $V(w) = 0.5(w(12 - w^2)$ (b) $V = 9$ so $h = 9/w^2$ and $A = 36/w + 2w^2$.

133. (a) $V = \pi r^2 h$, $A_B = \pi r^2$, $A_L = 2\pi rh$ (b) $A = 2\pi r^2 + 2\pi rh$.

134. $C = (0.02)4wh + 2*0.03w^2$, $V = w^2 h$, $V = 1000$, so $h = 1000/w^2$ and $C = 80/w + (0.06)w^2$.

135. (a) If the length L plus the girth G is less than 108, the volume can be increased by increasing L or G until L + G = 108. The volume is LA where A is the cross sectional area, and for a fixed perimeter of the rectangular cross section, A is a max if the cross section is a square. (b) L = 68 and V = 6800 cubic inches. (c) $L = 108 - 4w$ and $V = 4w^2 (27 - w)$, defined for $0 \le w \le 27$. (d) $V'(w) = 12w(18 - w)$ is 0 at w = 0, 18. V(0) = 0 , V(27) = 0. A is max at w = 18, for which L = 2sw = 36 inches.

136. (a) y = 15 and A = 150 (b) y = 5 and A = 150. (c) $y = 0.5(40 - x)$ and $A(x) = 0.5x(40 - x)$ (d) $A'(x) = 20 - x$ is positive for $0 \le x \le 20$ and negative for $20 \le x \le 40$. The total area is a max for x = 20, y = 10 feet.

137. (a) $8\pi = \pi r^2 h$, $h = 8r^{-2}$, $A(r) = \pi r^2 + 16\pi r^{-1}$ (b) A'(r) is 0 at r= 2 and $A(2) = 12\pi$.

138. (a) w = 8, h = 20, A = 160 (b) x = 0 and x = 6. (c) $w = 2x$, $h = 36 - x^2$, $A = 2x(36 - x^2)$. (d) A'(x) = 0 at $x = 2\sqrt{3}$ and has a max there, for which $w = 4\sqrt{3}$, h = 24 (e) w = 0 and h = 36 or w = 12 and h =0.

139. (a) $12 = 6x^2 h$, $h = 2x^{-2}$, $A(x) = 12x^2 + 12xh = 12x^2 + 24x^{-1}$ (b) A'(x) = 0 at x = 1. Min area at A(1) = 36.

140. (a) C(x) = 15x + 1500/x (b) C'(x) = 0 at x = 10. Min cost is $300.

141. $A(x) = 4x^2 + 8x^{-1}$, A'(x) = 0 at x = 1, min for x = 1, h = 4/3.

142. Cost = 120 = 4x + 5y, so y = 24 - 4/5 x , $A = x(24 - \frac{4}{5} x)$, A'(x) = 0 at x = 15 max area for x = 15 and y = 12.

143. (The volume should be 10 in the statement of the problem)(a) A(L) = 5L + 20/L + 4 (b) A'(L) = 0 for L = 2. The area is min for L = 2 , h = 1.

144. $C(x) = 3x^2 + 48x^{-1}$ C'(x) = 0 at x =2. Min cost for x = 2 y = 3/2.

145. $V = \pi r^2 h$, $V(r) = \frac{1}{2} \pi(12r - r^3)$, V'(r) = 0 at r = 2. The max volume is for r = 2, h = 2.

146. $A = 2\pi rh + 2\pi r^2$, $A = 4\pi/r + 2\pi r^2$, A'(r) = 0 at r = 1, h = 2, $A(1) = 6\pi$ is a min.

147. $A = (x - 2)(y - \frac{5}{2})$, y = 80/x since the area of the paper is 80. $A(x) = 85 - 160/x - \frac{5}{2} x$. A'(x) = 0 at x = 8. max area is for x = 8, y = 10.

148. $V = x^2 L$ for $0 \le x \le 11$, $2x^2 + 3xL = 600$. $V(x) = 200x - \frac{2}{3} x^3$. V'(x) = 0 at x = 10, V(10) = 1333 1/3 The volume is max for x = 10, h = 13 1/3.

149. Let x be the length of the side of the corral opposite the stable and y its width. The length of fence $x + 2y + (100 - x) = 180$, so $y = 140 - x$ and $A = xy = 140x - x^2$, defined for $100 \le x \le 140$. $A'(x)$ is negative for $100 \le x \le 140$. The corral of max area is x =100 ft long and y = 40 feet wide.

150. 7, -15

151. Q(4) = 10, Q'(4) = 48.

152. Approximately -66

153. 16 liters/hour.

154. (a) 0.003 literes/sec (b) h = 245, V = 5.045 liters

155. P(x(t)) dollars. The derivative is $\frac{dP}{dx}\frac{dx}{dt}$

156. 0.3 ft/min.

157. (a) $\rho(30.5) = 13.5205$ grams per ml. (b) $\frac{d\rho}{dt} = 0.0012$ grams per ml per min.

158. $\frac{dP}{dt} = -0.006$ atm per hr

159. (a) $\frac{dy}{dx}(6) = 0.875$, $\frac{dy}{dt} = 6.125$ ft per sec. (b) $\frac{dy}{dx}(10) = -1$, $\frac{dy}{dt} = -9$ ft per sec.

160. V(30) = 500, $\frac{dV}{dh} = 40$ thousand gallons per ft. $\frac{dV}{dt} = 120$ thousand gallons per hour.

161. 72

162. -30

163. 120

164. 72

165. -70

166. 20

167. Approximately -2

168. 12000

169. 139

170. -100

171. f(1) approx. 3.5 , f'(1) approx. –2.1

172. $h(t) = 1000(1.2^{t/5})$

173. $G(t) = 9(11^{t/5})$

174. The relative change is 0.37. The growth factor is 1.37.

175. $P = 10000(1.1)^{t/3}$

176. (a) 6 (b) 8

177. $f(x) = 5(3^x)$ $f'(x) = 5\ln(3)(3^x)$

178. $y = e^{x/3}$ $y' = \frac{1}{3}e^{x/3}$

179. $R(t) = 95 - 5(t - 1)$

180. $A(t) = 20 - 10t$, $B(t) = 20(0.5)^t$

181. (a) $p(h) = 1035(0.5)^{h/5.2}$ (b) $p'(10.4) = -34.49$ grams per sq cm per km

182. $P = 500(2^{t/3})$, $\frac{dP}{dt} = \frac{500}{3}\ln(2)(2^{t/3})$

183. (a) 2 half-lives, which is 9×10^9 years. (b) -1.925×10^{-10} grams per year.

184. L(48) = 1.25 mg per liter, decaying: -0.054 mg per liter per day.

185. N(20) = 9000, $N(t) = 1000(3^{t/10})$, N(0) =1000 bacteria N'(0) = 110 bacteria per hour.

186. The highest point on the curve is at x = ln(2). y = 1 + 2ln(2).

187. The inflection point is at x = 2, $y = 2e^{-2} - e^{-2}(x - 2)$.

188. $2e^{-6}$

189. $16e^{-4}$

190. $A = 2xe^{-x^2}$ The rectangle of max area has corners at $x = \pm 0.5\sqrt{2}$ on the x -axis. Max area of $\sqrt{2}\, e^{-1/2}$.

191. $r = \sqrt{13}$, $\sin(\theta) = 2/\sqrt{13}$, $\cos(\theta) = -3/\sqrt{13}$, $\tan(\theta) = -2/3$

192. x = 5.242, y = -2.049

193. (a) $h = 3\sin(\theta)$ (b) $\frac{dh}{dt} = 0.3$ feet per min

194. $\varphi + \theta = \pi/2$, $\frac{d\varphi}{dt} + \frac{d\theta}{dt} = 0$, $\frac{d\varphi}{dt} = -0.2$ rads per min.

195. (a) $\varphi = \cos^{-1}(\frac{1}{3}h)$ (b) $\frac{d\varphi}{dt} = 1/\sqrt{5}$ rads per min.

196. (a) frequency = 60 cycles per sec. (b) $V'(45) = 26400\pi\cos(5400\pi)$

197. $\frac{dA}{d\theta}(\pi/8) = 25\sqrt{2}$ $\frac{dP}{d\theta}(\pi,8) = -10\sin(\pi/8) + 10\cos(\pi/8)$

198. $A = 50\tan(x)$, $\frac{dA}{dx} = 50\sec^2(x)$.

199. Let x be the distance from the base of the ladder to the wall and θ the angle between the ladder and the ground. $\cos(\theta(t)) = \frac{1}{13}x(t)$. The angle is decreasing 1/6 rad/sec.

200. The top is moving -0.6cos(0.4) ft per sec and the base is moving 0.6sin(0.4) ft per sec.

201. -9.7163 meters per rad.

202. -1/8 rads per sec.

203. $\frac{d\theta}{dt} = \frac{1}{10}\cos^2(\theta)$

204. (a) 200 miles east of Ozona (b) His distance east of Ozona at t =6 equals his distance at t =0 plus the sum of the signed areas of the rectangles.

205. (a) 100 gallons (b) 350 gallons (c) 50 gallons (d) 100 gallons at t = 70. The final volume is the initial volume plus the sum of the signed areas.

206. (a) 18400 grams (b) The total weight equals the sum of the signed areas.

207. (a) $V = 86\pi$ cubic feet. (b) The volume equals the area of rectangles.

208. The average maximum temperature equals the sum of the signed areas, all divided by 7.

209. 500 feet

210. (a) Weight = 688 kilograms (b) Average density = $114\frac{2}{3}$ kg per meter

211. (a) a = 4, b = 9 (b) V = 69 cubic meters

212. Average value = $7\frac{1}{7}$

213. (a) 26 miles (b) Average velocity = 5.2 miles per hour.

214. (a) 2.63, –0.74 (b) 79.2 inches. (c) Average rainfall in 1881 through 1885 = 15.84 inches per year, which is 0.94 inches more than the average for 1970 through 1982.

215. (a) V = 582000 cubic feet. (b) a = 900, b = 3600, c = 6400. (c) The sum of the areas of the 3 rectangles equals the volume of the building.

216. (b) V= 360π cubic inches. (c) Weight = $0.273(360\pi) = 308.76$ pounds.

217. (a) 23360 million barrels, 21900 million barrels. 23360 - 21900 = 1460 million barrels.

218. Total kilowatt hours = 1.033×10^{11} kilowatt hours. Average kilowatts = 1,071,818 kilowatts

219. Average pressure = 110 pounds per square inch.

220. V(3) = 325, For t > 3, V(t) = 325 - 50(t - 3), t = 8.

221. 3

222. 549

223. Left Riemann sum = 150, right Riemann sum = 125.

224. 10000

225. f(2) = 3

226. $W(x) = \frac{1}{2}x^6 + \frac{3}{2}$

227. 175

228. W(0) = -3

229. G(4) = 3

230. 50

231. (a) 166.67 dollars (b) after 5 hours

232. (a) 5 feet per second (b) t = 16 seconds (c) 144 feet

233. (a) t = 5 (b) Distance = 215 nautical miles.

234. t = 4 minutes.

235. 171 cubic feet

236. (a) -3 feet per second^2 (b) 26.25 feet

237. At t = 6 the plane is 426 miles north of its position at t = 0.

238.(a) At t = 1 it is -4 feet west of its position at t = -3 or 4 feet east. (b) 8 feet toward the east and 4 feet toward the west or 12 feet total distance traveled.

239. Total distance = 13.17 feet

240. $k = 27$

241. $t = 2$

242. Weight = 5 kilograms Moment about $x = 0$ is 7 meter-kilograms Center of gravity = $\frac{7}{5}$ meters

243. Weight = $\frac{3}{4}$ pounds Moment about $x = 0$ is $\frac{3}{7}$ foot-pounds Center of gravity = $\frac{4}{7}$ feet.

244. Length for $0.1 \le x \le 1 = 99.061$, length for $1 \le x \le 100 = 99.309$. The part for $1 \le x \le 100$ is longer.

245. Area = $\frac{1}{2}$ square yard. Cost = \$2.50

246. Area = $\frac{6}{5}$ square meters. Weight = 18 kilograms

247. $s(t) = \frac{1}{6} t^3 + 3t - 2$

248. $s(t) = \frac{1}{4} t^4 + 4t + 5$

249. $s(t) = 5t + t^{-1} - 6$ meters, $a(t) = 2t^{-3}$ meters per minute^2

250. (a) 1 second (b) -48 feet per second.

251. $a(t) = 50$, $v(t) = 50t + 25$, $s(t) = 25t^2 + 25t + 100$

252. 144 feet

253. 4.5 seconds

254. $v(0) =$ feet per second

255. $\frac{4}{3} b^{3/2}$

256. $\frac{1}{2} hb$

257. $b = 3$

258. (a) Tangent line is $y = x$ (b) Area = $\frac{1}{3}$

259. (a) $\frac{1}{6}\frac{1}{m^3}$ (b) infinity

260. (a) $\frac{4}{3}$ $k^{3/2}$ (b) $A'(k) = 2\sqrt{k}$ which is the width of the top of the region since it extends from $-\sqrt{k}$ to $\sqrt{k}$.

261. k = 7.

262. (a) The solid has the shape of a curved horn with its tip at the origin in the xy plane and with a circular end. (b) $V = \frac{1}{60}\pi$ cubic feet.

263. Volume = $\frac{1}{7}$

264. $A(x) = 2(\sqrt{x} - x^2)^2$, $V = \frac{9}{35}$

265. $A(x) = \frac{1}{2}\pi(1 - x^2)^2$, $V = \frac{8}{15}\pi$

266. V = 27

267. 2,531,250 cubic meters

268. one approximation is 7,000,000 cubic feet.

269. $A(x) = (2 - 2x^3)^2$, $V = \frac{18}{7}$

270. $V = \frac{1}{3}\pi k^6 = 243\pi$, so k = 3

271. $A(x) = \pi[\frac{1}{2}(x^2 - x^3)]^2$, $V = \frac{1}{420}\pi$

272. $A(x) = (x^2/\sqrt{2})^2$, $V = \frac{16}{5}$

273. $A(x) = \frac{1}{4}\sqrt{3}(4 - x^2)$, $V = \frac{8}{3}\sqrt{3}$

274. $A(x) = 2\pi x(mx - x^2)$, $V = \frac{1}{6}\pi m^4$

275. $A(x) = \frac{1}{4}\sqrt{3}(x - x^4)^2$, $V = \frac{1}{36}\sqrt{3}$

276. $A(x) = \pi(x^{1/4})^2$, $V = \frac{2}{3}\pi k^{3/2} = 12\pi$, so $k = (18)^{2/3}$